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The Two-Region Fission-Electric Cell Reactor

Jerome L. Shapiro



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ABSTRACT

A modification of the Fission-Electric Cell Reactor is examined from the standpoint of reactor physics. This modification involves the insertion of a non-power-producing region, in order to maintain nuclear criticality. The goal is to increase the over-all efficiency by permitting thinner fuel layers in the electric-cell region.

A simplified, highly idealized model is utilized to make rapid calculations of a survey nature. This uses age-diffusion theory for the neutron transport. It also assumes that the added region is vanishingly thin, and is both an absorber of thermal neutrons and a source of fast neutrons. Then a multi-group, multi-region model is employed to check the accuracy of the simple model, and to give detailed power distribution data for one design problem.

Applied to a small (5-ft D) beryllium-moderated reactor, this calculational technique indicates that significant increases in over-all efficiency are achievable in the two-region reactor.

I. INTRODUCTION

A Fission-Electric Cell Reactor (FECR) would utilize as its power source the kinetic energy of fission fragments which escape from a fuel-bearing cathode (Refs. 1 and 2). Therefore, a most important factor in the efficiency of the FECR would be the fuel-layer thickness. This feature raises an unusual problem in reactor physics; namely, how the fuel should be distributed to maximize the efficiency, integrated over the life of the reactor.

In a conventional reactor, the fuel is distributed so as to flatten the power distribution. Additional fuel, when desired, is simply added in the form of increased initial fuel density, and its presence is balanced by the use of shim rods or burnable poisons, or both. This technique usually has a small depressing effect on the over-all reac-

tor performance because the ratio of peak power to average power is increased. There is no noticeable decrease in energy-transfer efficiency caused by the extra fuel.

In the FECR, however, the addition of fuel to the layer would cause a severe drop in efficiency because of a decrease in the average fission-fragment escape probability. A system which adds fuel continuously to compensate for burnup would, of course, solve this problem. Short of this, it is reasonable to question whether it is possible to distribute the fuel in such a manner that the decrease in efficiency is not as drastic as that of increased fuel-layer thickness. This possibility is the subject of this Report.

Because of the complexity of the situation, it was not feasible to answer the question or even to approach it in a general manner. Therefore, it was necessary to choose some input data rather arbitrarily. It should be emphasized that while specific reactor sizes and nuclear constants were used, no attempt was made to optimize the design of this reactor. It is sufficient at this point to show the effects of various fuel-placement schemes on the overall efficiency, and to demonstrate the validity of the calculational methods.

At the outset, a highly idealized model was developed to provide an analytical ground work for the inevitable computer formulation. The results of this phase, although not quantitatively accurate, were sufficient to indicate the usefulness of more detailed calculations, and they also gave valuable insight into the problem, effectively reduc-

ing the number of computer operations required. In the second phase, the AIM-6 code (Ref. 3) was used. This permitted the solution of an 18-group, multiregion model in one (radial) dimension. Various search options were utilized to determine the major design parameters.

It was assumed at all times that any heat generated in any part of the reactor would be dumped. In other words, although considerable energy may be produced in a region other than the fission-electric cells, this was considered to be waste. The ratio of the energy released in the reactor without added fuel to that of the reactor with fuel added (with output electric power assumed constant) is termed the Efficiency Reduction Factor (ERF). Clearly, the best design is the one with the highest ERF. The following analysis is concerned solely with the evaluation of this parameter.

II. ANALYSIS

As a first approach, consider a bare, cylindrical, homogeneous reactor loaded with sufficient fuel to be initially critical. As power is generated, however, the reactor will become subcritical because of fuel burnup. To maintain a constant power level, fast-neutron sources are added. The source intensity must increase as fuel is used up. Also, the flux shape will vary due to the contribution of the sources. These hypothetical sources are assumed to occupy no volume, and the sources are defined so that for each η^{25} fast neutrons produced, one thermal neutron is absorbed.

Age theory will be used to determine the thermal-flux distribution. Then the multiplication, defined as the ratio of fission-neutron production rate to source-neutron production rate, will be calculated. From the multiplication, the ERF will be estimated, assuming that the sources release the same total energy per neutron as would be produced in fission.

The effects of three different source distributions will be compared. They are:

1. A line on the axis
2. A cylindrical surface on the outer boundary of the reactor
3. A uniform distribution throughout the reactor

A. Theory

1. Line Source

The problem is to describe the thermal flux in a finite bare cylinder with a line at the center, which is both a fast-neutron source and a thermal-neutron sink. Separation of radial and axial variables is assumed. The radial part of the problem can then be stated:

$$D\nabla^2\phi(r) - \Sigma_a\phi(r) + S\frac{e^{-r^2/4\tau}}{4\pi\tau} - \frac{\delta(r)}{2\pi(r)}S\frac{\Sigma_a^{25}}{\nu\Sigma_f^{25}} + q_f(\tau, r) = 0 \quad (1)$$

$$\nabla^2 q_f(\tau', r) = \frac{\partial q_f}{\partial \tau'} \quad (2)$$

$$q_f(0, r) = \nu\Sigma_f\phi \quad (3)$$

$$\phi(R) = 0 \quad q_f(R, \tau') = 0 \quad (4)$$

The use of the third term in Eq. (1) is based on the assumption that $R^2 \gg 4\tau$. The solution may be found with the aid of Hankel transformation:

$$\phi(r) = \frac{S}{\pi R^2 \nu \Sigma_f} \sum_i \frac{J_0(\psi_i r)}{[J_1(\psi_i R)]} \frac{k_i}{1 - k_i} \left[1 - \frac{\exp[(\psi_i^2 + B_z^2)\tau]}{\eta} \right] \quad (5)$$

where

$$k_i = \frac{k_\infty \exp [-(\psi_i^2 + B_z^2) \tau]}{1 + L^2 (\psi_i + B_z^2)} \quad (6)$$

and

$$J_0(\psi_i R) = 0 \quad (7)$$

Defining the multiplication as

$$M = \frac{\text{fission-neutron production rate}}{\text{source-neutron production rate}} \quad (8)$$

we see that

$$M = \frac{2\pi v \Sigma_f}{S} \int_0^R \phi(r) r dr \quad (9)$$

and, therefore,

$$M = \sum_i \frac{2}{\psi_i R J_1(\psi_i R)} \frac{k_i}{1 - k_i} \left[1 - \frac{\exp [(\psi_i^2 + B_z^2) \tau]}{\tau} \right] \quad (10)$$

As the reactor fuel burns out, k_i will decrease from its initial value of unity, and M will decrease from its initially infinite value. The source strength S will be increased to maintain a constant fission power, and the consequent ERF will be simply

$$\text{ERF} = \frac{M}{1 + M} \quad (11)$$

2. Cylindrical External Source

The problem of the cylindrical external source can be treated in a manner similar to the problem of the line source. However, the thermal flux caused by the external source must first be determined:

$$\nabla^2 q(r, \tau') = \frac{\partial q}{\partial \tau'} \quad (12)$$

$$q(r, 0) = S \delta(r - R) \quad (13)$$

$$\frac{\partial q}{\partial r} + \frac{q}{2d} = 0 \quad (r = R, \tau > 0) \quad (14)$$

The last Equation describes the condition of no inward current. Again, applying the Hankel transform technique, the solution to this problem is

$$q_s(r, \tau') = \frac{2S}{R} \sum_i \frac{\psi_i^2 \exp(-\psi_i^2 \tau') J_0(\psi_i r)}{\left[\psi_i^2 + \left(\frac{1}{2d} \right)^2 \right] J_0(\psi_i R)} \quad (15)$$

where

$$\frac{\psi_i R J_1(\psi_i R)}{J_0(\psi_i R)} = \frac{R}{2D} \quad (16)$$

This result may now be used to determine the radial flux shape:

$$D \nabla^2 \phi(r) - \Sigma_a \phi(r) + q_s(\tau, r) + q_f(\tau, r) = 0 \quad (17)$$

$$\nabla^2 q_f(\tau', r) = \frac{\partial q_f}{\partial \tau'} \quad (18)$$

$$q_f(0, r) = v \Sigma_f \phi(r) \quad (19)$$

$$\frac{\partial q_f}{\partial r} + \frac{q_f}{2D} = 0 \quad (r = R, \tau > 0) \quad (20)$$

The solution to these Equations is

$$\phi(r) = \frac{2S}{R v \Sigma_f} \sum_i \frac{k_i}{1 - k_i} \frac{(\psi_i R)^2}{\left(\frac{R}{2D} \right)^2 + (\psi_i R)^2} \frac{J_0(\psi_i r)}{J_0(\psi_i R)} \quad (21)$$

where ψ_i is defined by Eq. (16). The multiplication M is now

$$M = \frac{R}{2D} \sum_i \frac{k_i}{1 - k_i} \frac{2}{\left(\frac{R}{2D} \right)^2 + (\psi_i R)^2} \quad (22)$$

The ERF can also be calculated as before (Eq. 11). There is an important feature that distinguishes the external source from the internal source. In principle, one can always (mathematically, if not physically) maintain power level using an internal source by adding increasing amounts of fissionable material to the source line. In the external cylinder case, no matter how much fissionable material is added to the outside, the contribution of the source is limited by the leakage rate from the core. Thus, it is possible that at some point in time there will be insufficient leakage from the core to activate the source cylinder enough to sustain power. This was the case in the reactor chosen as an example.

3. Uniformly Distributed Source

This distribution is the furthest from reality because of the assumption that sources can be added uniformly over the reactor, as required, to compensate for burnup. However, the analysis is simple and, in addition, something can be learned from comparison of the results with the previous cases.

Obviously, in this case, the higher spatial modes of the flux will not be excited. The multiplication should then be, simply:

$$M = \frac{1}{1-k} - 1 = \frac{k}{1-k} \quad (23)$$

while the ERF is again the same as in Eq. (11).

B. Calculations

The reactor model chosen is a beryllium-moderated cylinder, 5 ft in diameter and 5 ft high, with electrical-power output of 1 Mw. This corresponds to an early concept of the FECR. The following nuclear constants were assumed at operating temperature:

$$\tau^{Be} = 89.6 \text{ cm}$$

$$\sigma_a^{25} = 296 \text{ b}$$

$$\sigma_f^{25} = 248 \text{ b}$$

$$\sigma_a^{Be} = 0.01 \text{ b}$$

$$D^{Be} = 0.497 \text{ cm}$$

The core geometry chosen was:

$$H = 150 \text{ cm (total reactor height)}$$

$$R_1 = 1 \text{ cm (fission-electric cell cathode radius)}$$

$$R_2 = 1.5 \text{ cm (anode radius)}$$

$$t^{25} = 12.6 \text{ mg/cm}^2 \text{ (fuel thickness)}$$

The fuel cells were arranged in a close-packed, triangular lattice.

From these parameters, the following parameters were calculated:

$$\text{Void fraction} = 0.505$$

$$N^{25} = 2.6 \times 10^{19} \text{ atoms/cm}^3 \text{ (at } t = 0)$$

$$N^{Be} = 5.9 \times 10^{22} \text{ atoms/cm}^3$$

and

$$\tau = 366 \text{ cm}^2$$

$$D = 1.005 \text{ cm}$$

$$\Sigma_a^{Be} = 0.00059$$

$$\Sigma_a^{25} = 0.00770 \text{ (at } t = 0)$$

$$\Sigma_f^{25} = 0.00645 \text{ (at } t = 0)$$

These data were used to find the "just critical" radius, utilizing

$$1 = \frac{\nu \Sigma_f}{\Sigma_a} \frac{e^{-B^2 \tau}}{1 + L^2 B^2}$$

and

$$B^2 = \left(\frac{\pi}{H}\right)^2 + \left(\frac{2.405}{R}\right)^2$$

This produced $R = 77.1 \text{ cm}$.

The variation in Σ_f and Σ_a with time was treated by estimating the average burnup rate of U^{235} as

$$\begin{aligned} & \frac{\text{electrical power}}{\text{total efficiency}} \times \frac{\text{fissions/w-sec}}{\text{volume}} \times \frac{\Sigma_a^{25}}{\Sigma_f^{25}} \\ &= \frac{10^6}{0.031} \times \frac{3.1 \times 10^{10}}{\pi (77.1)^2 150} \times \frac{296}{248} \\ &= 4.26 \times 10^{11} \text{ atoms/cm}^3\text{-sec} \end{aligned}$$

Neglecting spatial burnup considerations,

$$\frac{\Sigma^{25}}{\Sigma_{t=0}^{25}} = 1 - 0.00142t$$

where t is in days. Results are shown in Table 1.

Table 1. Effect of fuel burnup on nuclear parameters

t	$1 - 0.00142t$	Σ_f	Σ_a	L^2	$\nu \frac{\Sigma_f}{\Sigma_a}$
0	1	0.00645	0.00829	121.2	1.945
100	0.858	0.00553	0.00720	139.5	1.922
200	0.716	0.00462	0.00610	164.7	1.894
300	0.574	0.00370	0.00501	200.8	1.847
400	0.432	0.00279	0.00392	256.5	1.780

C. Results

The radial power distributions after different operating periods with an axial line source are shown in Fig. 1. These were calculated using 40 modes. Due to the line sink at the center, this technique does not give very rapid flux convergence at small radii. However, the multiplication factor converges much more rapidly, and this is the more important characteristic here.

Radial power distributions for the external cylinder source are shown in Fig. 2. With the model used here, this system will be self-supporting for less than 100 days.

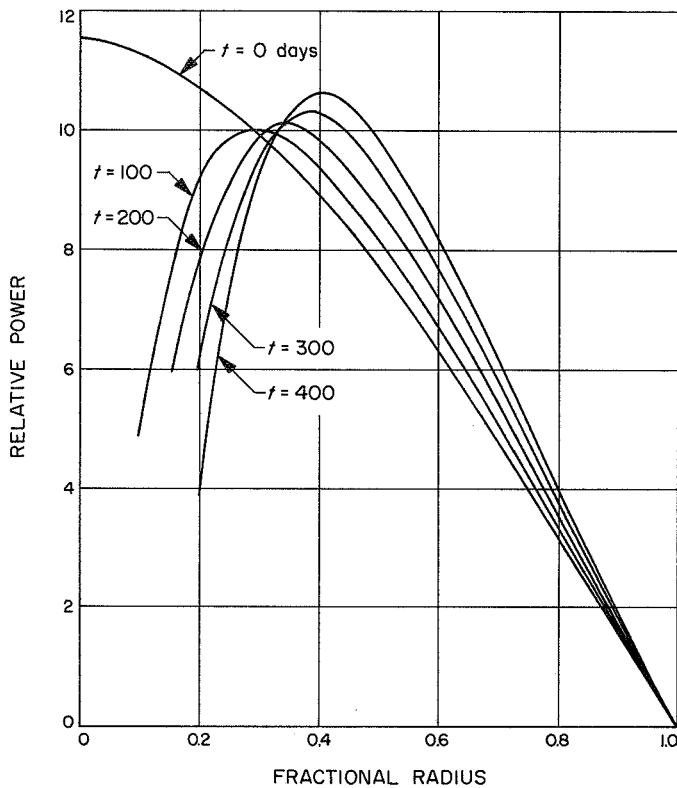


Fig. 1. Radial power distribution with central line source

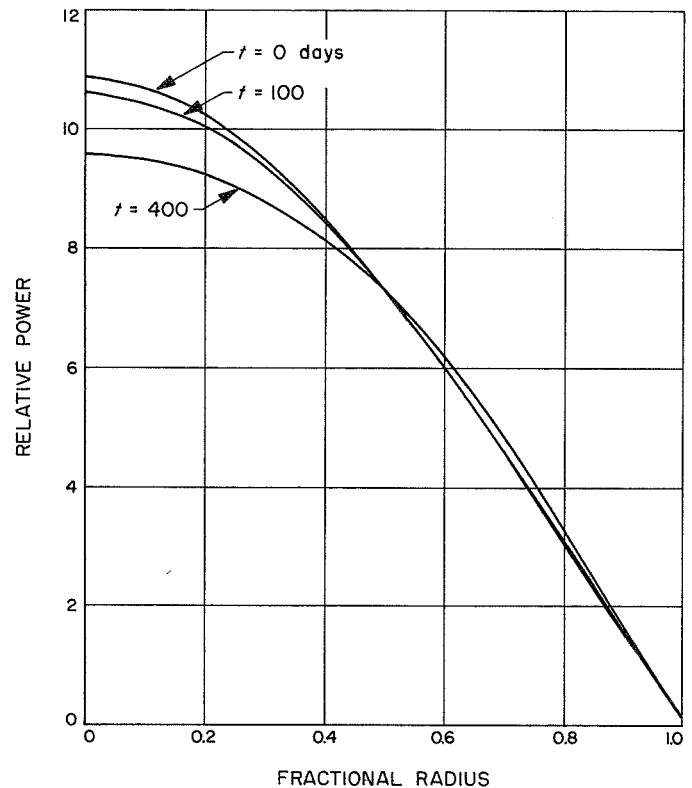


Fig. 2. Radial power distribution with outer cylinder source

In an actual case, however, if an outer fission blanket were used, a reflector material would most certainly be added to increase the effectiveness of the blanket.

Figure 3 shows the effect of burnup on the ERF for the various models assumed. The effect of adding fuel directly to the cathode is also shown for comparison. Some interesting features may be noted. First of all, the advantage of adding fuel to any place other than directly on the cathode is obvious. Further, it can be seen that within the limits of the model, the best arrangement would be to have a thin cylindrical fuel region inserted somewhere in the core. This conclusion results from the observation that the ERF for the uniform source distribution is higher than for the central line or outer source. This implies that a three-region reactor, with inner and outer fission-cell regions, separated by a high fuel-density driver annulus, may be the best from the standpoint of neutron physics. However, due to the increased complexity, the three-region system was not examined.

These results were sufficiently encouraging to warrant the more detailed calculations presented in the following Section.

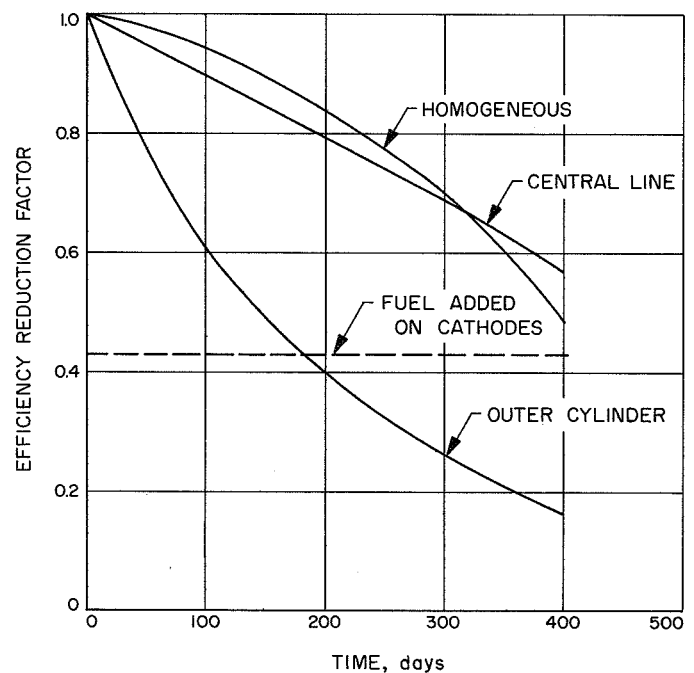


Fig. 3. Effect of burnup on efficiency reduction factor

III. MULTIGROUP-MULTIREGION CALCULATIONS

A. Input Data

The material densities used here are the same as those calculated previously. For the fission-cell region they are:

$$N^{Be} = 5.9 \times 10^{22} \text{ atoms/cc}$$

$$N^{25}(t = 0) = 2.59 \times 10^{19} \text{ atoms/cc}$$

$$N^{25}(t = 400 \text{ days}) = 1.12 \times 10^{19} \text{ atoms/cc}$$

The 18-group cross-section library in the AIM-6 code was used without alterations.

The radius chosen was 78 cm. The first operation was a transverse-buckling search to determine the critical height at $t = 0$. The resulting buckling was 0.0011 cm^{-2} , equivalent to a reactor height of 95 cm. This value was used in the remainder of the calculations.

B. Variation of Driver Radius

The driver region should be a small, high-leakage section; therefore, no moderating material was placed in it. The region was assumed to be an axial cylinder composed of Na (coolant) and U^{235} only. The driver radius used was 6, 10, 14, and 18 cm, in turn. Then, the necessary amount of U^{235} in the driver to make the reactor critical at $t = 400$ days was found. The results are shown in Table 2. Included in Table 2 is the ratio of average power in the driver to average power in the core as well as the ERF (at burn out). Clearly, there is little need to make the driver larger than 14 cm. When the radius is reduced below 10 cm, the average power density in the driver increases sharply, as does the fuel concentration. Therefore, the driver radius should be between 10 and 14 cm. For the remainder of the calculations the radius was assumed to be 14 cm.

Table 2. Effects of driver-radius variation

Radius	Fuel volume concentration	Average driver-power density	ERF
		Average core-power density	
6	24	66	0.72
10	12	31	0.66
14	8.3	23	0.57
18	7.0	19	0.49

C. Fuel Distribution in Driver

The radial power distribution in a two-region reactor with uniform fuel distributions in each region is shown in Fig. 4. In order to flatten the driver power distribution, it was divided into 7 regions of different fuel densities. The relative fuel densities in each portion are approximately inversely proportional to the power density shown in Fig. 4.

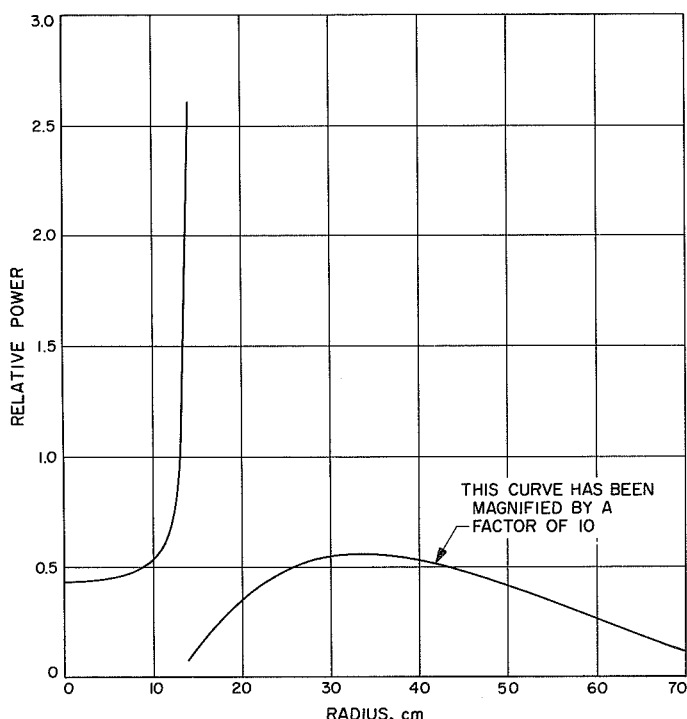


Fig. 4. Radial power distribution in a two-region FEER with uniform fuel concentration in each region

D. Power and Flux Distribution

Figure 5 shows the radial power distribution, thermal-flux distribution, and epithermal-flux distribution (3.4–17 kev). In this case, a 2-cm thick pure-beryllium region was added between the driver and fission-cell regions. Hopefully, this additional region could be used, with poison rods, to control the reactor by limiting the passage of neutrons between the two main regions.

E. Median Fission Energy

One calculation was made in which the median fission energy at each mesh point was calculated. The result is displayed in Fig. 6.

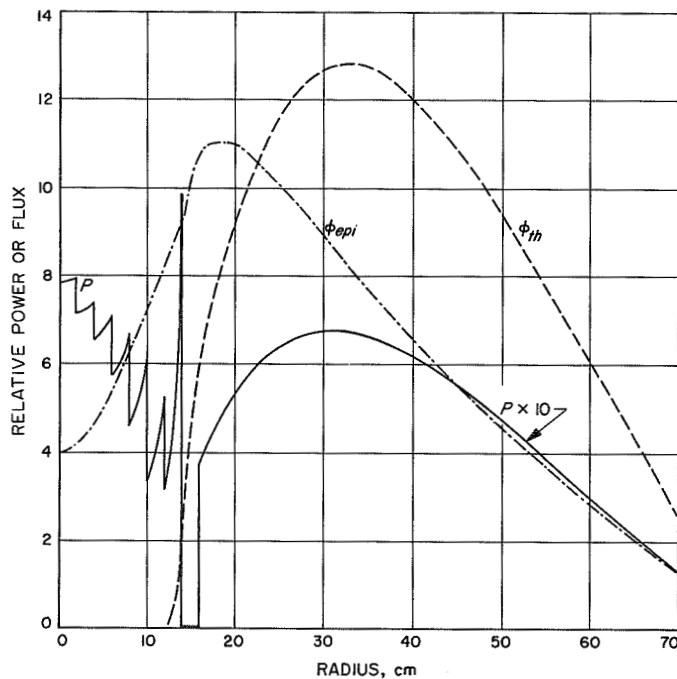


Fig. 5. Radial flux and power in a clean FECR

F. Reactor-Core Reference Parameters

The simple age-diffusion model, while adequate for survey purposes, may not be sufficiently accurate for predicting power distributions and controllability. Therefore, the 9-region multigroup model was applied to a specific reactor. The driver fuel-concentrations, for this case, are shown in Table 3.

For the clean core, with boron added to the control region, the calculation results in $ERF = 0.72$ and in an average power-density ratio of 12. At burn out, we find $ERF = 0.59$ and an average power density ratio of 21. Note that these latter values agree very well with the 14-cm radius values in Table 2, which were calculated with a two-region model.

A calculation of the effect of loading the control region with boron indicated that from 25 to 30 dollars of negative reactivity could be inserted in this manner.

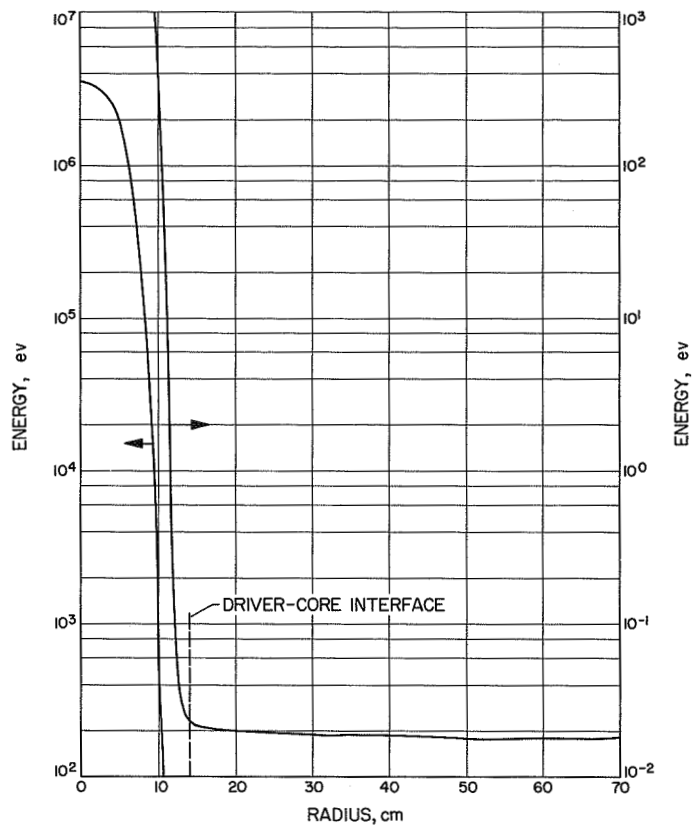


Fig. 6. Spatial distribution of median fission energy in a two-region FECR

Table 3. Fuel distribution in the driver of the reference core

Region	Volume % concentration,
1	14.6
2	14.2
3	13.6
4	12.5
5	10.7
6	7.3
7	2.1

IV. CONCLUSIONS AND OBSERVATIONS

Two rather important conclusions can be drawn from the results of this study. First, it is clear that the uniform distribution of fuel on fission-electric cell cathodes may be far from optimum. Second, the over-all ERF appears to be relatively insensitive to variation of the fuel distribution within the driver region. This may be of value in simplifying future calculations and core designs.

Although the simple Fermi-age approach gave reasonable results and may be used to show trends, the agree-

ment with the multigroup results is not good except for the large-radius driver region. Indeed, even this limited agreement may be merely fortuitous.

In the case considered in this Report, the fuel assumed to be on the cathodes is the amount necessary for clean criticality. This was an arbitrary choice, however, and raises the obvious question whether or not further advantage may be gained by displacing still more of the fuel to an interior non-power producing region. This possibility is being explored further.

NOMENCLATURE

B_z^2	axial buckling	S	neutron source strength
D	diffusion coefficient of neutrons	t	time
k_i	criticality factor associated with i th neutron flux mode	δ	Dirac delta function
k_∞	infinite medium multiplication factor	η	average number of neutrons released per absorption in U^{235}
L	diffusion length of thermal neutrons	σ	microscopic cross section
N	atomic density	Σ_a	macroscopic-absorption cross section
q_f	slowing-down density of fission neutrons to thermal energy	Σ_f	macroscopic-fission cross section
q_s	slowing-down density of source neutrons to thermal energy	ν	average number of neutrons released per fission
		τ	age to thermal energy
		ϕ	neutron flux

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